RESECTION

The last newsletter focused on the basic principles of **Resection** and the **least squares methods** that are applied in this setup method. This newsletter is again about **Resection** but gives much more detail about the applied mathematical models.

As explained in the last newsletter the **Resection** method allows some or all of the following parameters to be computed and set:

- ?? 2D position of station
- ?? Height of station
- ?? Orientation
- ?? Scale factor

Computation of these parameters can be done using either the regular **least squares adjustment** or the so-called **robust calculation**. Before looking in detail at these methods some important points common to both methods should be explained.

TARGET POINT ACCURACY

A resection uses measurements of up to ten target points to calculate the parameters mentioned above. These target points may be 3D, 2D or height-only points. The estimated accuracy (that is, the "reliability") of the target points can be defined in the **SETUP Configuration** panel:



These estimated accuracies are then used in the least squares adjustment calculation to construct realistic weight information for the observations.

CALCULATIONS

The principle aim of the calculations is to use all available data to determine the results whilst also identifying any possible errors. The calculations are actually split into two parts – the computation of **Easting**, **Northing**, **Orientation** and the **Scale factor** and the computation of the **Height**. Both routines compute residuals and test for outliers.

Note, the scale factor can only be determined if at least one distance measurement has been made to a target point. Remember, to compute the scale factor, it is necessary to select **Yes** at the **Use Scale** prompt in the **SETUP Configuration** panel.

LEAST SQUARES / ROBUST ADJUSTMENT

It is possible to choose between using the **Least Squares** and **Robust** calculation methods in the **SETUP Results** panel.

<u>13:38</u> SETUP	۲	${}_{\text{std}}^{\text{IR}}$ I		` 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Results (L	east S	quare	<u>s)</u>	×
Stn Coords	Sigma S	Stn Coo	de Plot	
Station ID	:			0001
No. of Poi	nts:			3
Set	:	Ε,	N, Ht,	0ri 🜗
Instrument	Ht:		Ó	.000 m
Stn Eastin	g :		50	. 000 m
Stn Northi	ng :		50	. 000 m
Stn Height	ँ :		100	. 000 m
New Azimut	h :		150.	0001 g
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SET	ROBS	T INF	O SUR	Y PAGE

Simply press **F3(ROBST/LSQRS)** to toggle between the two methods – the results are immediately recalculated and updated.

WEIGHTING

Both methods actually use least squares computations but for the **Robust** method a modified form of the general principle of least-squares adjustment is applied – in simplest terms, the least-squares technique is adopted with "robust weighting".

This difference is explained in more detail below.

Least Squares Method: The weight (or standard deviation) which is assigned to each observation is composed of a constant and a variable part.

The constant part depends on the pre-defined angular and EDM accuracies of the instrument.

The variable part is computed from the accuracy of the target points (as defined earlier in the **SETUP Configuration** panel) and the distance between target point and station.



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Robust Method: Rather than applying weights to observations as described above, weights are calculated depending on the fit between the observed and the computed value for an observation. Therefore, observations which are in good agreement with the computed values are awarded higher weights relative to those which have large differences between observed and computed values.

The advantage of the robust weighting is that it enables good results to be obtained even if errors are within the data. Bad data is effectively "de-weighted" and therefore has little or no influence upon the results.

Note, if all measurements are of high quality the results for the robust method will be consistent with the results of the least squares method.

DETERMINATION OF PROVISIONAL VALUES

Using the available target point coordinates and measurements, various geometrical routines are used to compute "provisional" easting/northing co-ordinates from unique combinations of the data. Various checks are applied to ensure that "chance" solutions are excluded.

Having identified a group of results which contain the best potential provisional values, the **median** values are determined. Theoretically, any result within this group should be suitable, however, a median value is taken to add extra confidence to the selection. This also helps to avoid selection of a value, which is not in agreement with the rest of the values.

The procedure for determining provisional values for height is more straight forward as only one observation needs to be considered at a time. Again the **median** height value from the set of provisional values is selected as the best provisional value to use.

The approach described for determining the provisional values is very reliable but is limited by the available redundancy and the percentage of errors in the data. As the redundancy falls and the percentage of errors increases so the likelihood of obtaining good provisional values decreases.

ITERATING TO THE RESULT

Once weights for all observations and provisional values for the results are available the least squares adjustment routines can be started.

When performing a resection computation up to five parameters may be determined - easting,

northing, height, orientation and scale factor. The calculation of the final results may require a number of iterations until the change in the computed parameters are no longer significant – this is when changes in the values are less than the set criteria listed below.

Parameter	Criteria
Easting	0.5mm
Northing	0.5mm
Orientation	2"
Scale factor	See below
Height	0.5mm

The scale factor criterion is a function of the positional accuracy of the target points and the maximum distance measurement. For example, if an accuracy of 10mm has been specified and distances up to 1000m have been measured, then the resulting scale factor must be determined to 10ppm.

PUTTING IT INTO PRACTICE - AN EXAMPLE

Below is an example to illustrate the effect of using the different methods

Imagine four target points arranged in a square with a side length of 100m. The TPS instrument is set up roughly in the centre and it is necessary to determine the coordinates of the setup using a resection.



Unfortunately for the surveyor, the chainman has set up the reflector at P2 on the wrong point - it has been setup over a point which actually has the coordinates (96,98,100).

The surveyor does not realise this and starts the resection routine and measures angles and distances to the 4 points.



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After measuring the 4th point he presses **F5(CALC)** to compute the resection and view the results.

Immediately after pressing **F5(CALC)** (and before the results are shown) a series of warnings inform the surveyor that the positional accuracy and orientation accuracy were not achieved. The surveyor acknowledges these warning messages and then the **SETUP Results** panel **Stn Coords** page view is shown.

14:24			ر لا	
Results (L	east Squ	uares)		×
Stn Coords	Sigma St	n Code∣F	lot	
Station ID	:		Statio	n1
No. of Poi	nts:			4
Set	:	E, N,	Ht, O	ri 💁
Instrument	Ht:		0.0	00 m
Stn Eastin	g :		51.7	82 m
Stn Northi	ng :		50.2	90 m
Stn Height	- :		100.0	00 m
New Azimut	h :	2	250.94	52 g
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SET	ROBST	INFO	SURVY	PAGE

In order to check the quality of the results the surveyor accesses the **Sigma** page view – it now starts to become obvious that something is wrong. The standard deviations and scale factor are far too big.

<u>14:26</u> SETUP	ST	D I	ີ ຊີ ຊີ	
Results (L	east Squ	ares)		X
Stn Coords	Sigma <u>[Str</u>	n Code Plo	t	
σ Easting	:		1.221	m 🔺
σ Northing	:		1.221	m
σ Height	:		0.000	m
σ Hz Orien	t :	0	.7363	g
Calc Scale	:	1.02	02504	L
Calc PPM	:	20	250.4	L 🚽
Set as	:	Geometri	ic PPľ	┣║╼
				a û
SET	ROBST	INFO SU	RVY P	AGE

The surveyor decides to investigate this in more detail and looks at the residuals of the individual observations by pressing **F4(INFO)** to access the **SETUP Additional Information** panel.

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SETUP	STD	T	z 🗢 🛽
Addition	al Informat	ion	×
Status P1	ot		
Point ID		Use	ΔHz [g]
P1		3D	-2.2443
P2		3D	1.8660
P3		3D	0.3932
P4		3D	-0.0149
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RECLC	USE R	EMOV	MORE PAGE

On initial inspection, the surveyor may think that since the dHz residual value to P1 is the biggest then there must be something wrong with the measurement to P1 or with the control coordinates of P1.

But looking closer it can be seen that the ! mark in the left-most column indicates that there may actually be something wrong with the measurements to **P2**.

It should be remembered that a gross error in any observation affects the residuals of all doservations - but it is not necessarily the observation with the biggest residual that contains the error.

The surveyor presses the **F5(MORE)** button which changes the right-most column to display the residuals for the dDist observations.

14:28		I	
Additional	Informat	ion	
Status Plot			
Point ID		Use	∆Dist [m]
P1		3D	-0.355
P2		3D	2.764
P3		3D	-2.423
P4		3D	0.039
			a បំ
RECLC	USE R	EMOV	MORE PAGE

It is now clearer to the surveyor where the error may lay...

But for now the surveyor presses **ESC** to return to the **SETUP Results** panel and changes to the robust calculation by pressing **F3(ROBST)**.



_14:30	👝 IR	Т	<u>``</u> !!	
SETUP	ST 🖤 ST	ъ т	_ r 🗸	> 🕘
Results (Ro	bust)			X
Stn Coords S	Sigma∫Str	n Code(Pl	ot	
Station ID	:	St	ation1	
No. of Poin	ts:		4	
Set	:	E, N, H	t, Ori	♠
Instrument	Ht:		0.000	m
Stn Easting	:		50.000	m
Stn Northin	g :		50.000	m
Stn Height	Č :	1	00.000	m
New Azimuth	:	25	0.0000	g
				a û
SET	LSQRS	INFO SL	JRVY P	AGE

The results (easting and northing of the station) are very different from the ones in the least squares results panel – and a look at the **Sigma** page view confirms this.

14:31 SETUP	🔮 I R STI	J	" "	
Results (Ro	obust)			×
Stn Coords S	Sigma <mark> St</mark> n	ı Code Plot		
σ Easting	:	0	0.011	m 🔺
σ Northing	:	0	0.011	m
σ Height –	:	0	.000	m
σ Hz Örient	: :	0.	0071	g
Calc Scale	:	0.999	9966	
Calc PPM	:		-3.4	
Set as	:	Geometric	; PPM;	╟╺
				аû
SET	LSQRS	INFO SUR	VY PA	GE

The standard deviations are much smaller and the calculated scale is more realistic.

The surveyor presses **F5(INFO)** again to view more detailed information about the residuals of the individual observations for Hz ...

14:32 SETUP	STD	Ι	` 🖺 🗖
Additional	Informat	ion	×
Status Plot			
Point ID		Use	ΔHz [g]
P1		3 D	-0.0001
? P2		3 D	3.7809*
P3		3 D	-0.0000
P4		3 D	-0.0000
			a û
RECLC	USE R	EMOV	MORE PAGE

and again press **F5(MORE)** to view the residuals for the distances...

14:32 SETUP	IR Std	Ι	` M 🗖
Additional	Informat	ion	×
Status Plot			
Point ID		Use	∆Dist [m]
P1		3D	0.000
P2		3D	5.596
P3		3 D	-0.000
P4		3D	0.000
			a û
RECLC	USE R	EMOV	MORE PAGE

There is now no doubt in the surveyor's mind as to which observation is at fault and can investigate further.

It can now be seen that due to the different weighting mechanisms used in the robust method, the wrong observation to P2 was detected and "de-weighted" to such an extent such that its influence on the result is practically zero.

In order to get a "clean" result the surveyor now has 3 options:

- Disable observation P2 for the calculation by pressing the F3(USE) button until Use is set to No and then press F1(RECLC). This keeps the observation in the database, but does not use it for the calculation of the result
- Completely remove the observation by pressing F4(REMOV). This also deletes the measurement permanently from the job.
- Return to SETUP Results panel and press F5(SURVY) to re-measure point P2 (assuming the chainman has managed to set the reflector over the correct point this time)

FINAL HINT

Once the surveyor had excluded the wrong measurement from the calculation, then both methods (least squares and robust) will deliver the same result.

This means that the two methods can be used to quickly check the "robustness" of the resection. Once in the results panel simply toggle between the two methods and check if the result changes. If the results are very similar then the resection result should be acceptable. **f** it does change then it maybe worth to investigate further and have a closer look at the residuals.



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